Quantum Mechanics

T. U. Solution 2078.

1) List the issues examplaining inadequacy of classical mechanics. How quantum mechanics becomes useful to give better solution. How Davisson-Germer experiment supported quantum mechanics? explain.

Solution

Classical mechanics based on the Newton's three law of motion (i) Laws of inertia (ii) Laws of force (III) Laws of action and reaction., totally germinated on the ground of absolute mass, absolute space and absolute time that could explain the motion of the body with non-relativistic speed. (i.e $v << c$).

The inadequacy of classical mechanics are as,

i) classical mechanics couldn't explain the stability of atoms.

ii) It couldn't explain the black body radiation.

III) It couldn't explain the observed variation of specific heat of solid.

IV) It couldn't explain the origin of discrete

(v) It couldn't explain a large number of phenomenon like photoelectric effect, compton effect, Raman effect etc.

2nd part

Quantum mechanics become useful and provides better solution in various ways.

(a) <u>Nanotechnology</u> :- Quantum mechanics provides the theoritical framwork for understanding and manipulating matter at the ~~nanoscence~~ nanoscale, leading to the development of <u>nanotechnology</u>. This has practical application such as electronics, material science, and medicine.

(b) ~~Q~~ <u>Quantum computing</u> : Quantum mechanics enable the development of quantum computers, which have the potential to perform certain calculation much faster than classical computers. Quantum <u>algorithms</u> can solves problems in ~~crop~~ cryptography, optimization and simulation more efficiently.

(c) <u>Quantum sensing</u> :- Quantum mechanics enables the development of highly sensitive sensors, such as quantum enhanced imaging and quantum sensors for precise measurement in fields like metrology, navigation and medical diagostics.

(d) Superconductivity:- Quantum mechanics explain phenomena like superconductivity where materials exhibit zero electrical resistance at low tempratures.

(e) Quantum metrology:- Quantum mechanics provides advanced technique for precise measurements, surpassing the limitations essential for applications in field such as time keeping, navigation, and fundamental physics research.

Overall, Quantum mechanics revolutionizes various fields by offering fundamentally new approaches and solution that out-perform classical methods in certain tasks.

3rd part

The Davisson Germer experiment provides significant supports for the principle of Quantum mechanics. This

Date
Page

experiment provides experimental evidence that electrons exhibit wave-like behaviour, supporting the wave particle ~~duality~~ duality. concept central to quantum mechanics. It demonstrated the need to embrace a probabilistic and wave based description of particles at the quantum level, which lead the ~~functi~~ fundation of modern quantum theory.

Date
Page

Q(1) OR

Discuss Ramsaur - Townsend effect in the of scattring of low energy electron by atoms of a nobal gas. Find transmission probability for electrons.

Solutions

Consider a square potential well of finite depth (V_0) as shown as figur. and width 'a'. Suppose a uniform beam of partical each of mass 'm' and having total positive energy $E>0$ in traveling from left to right parallel to +ve direction of x-axis

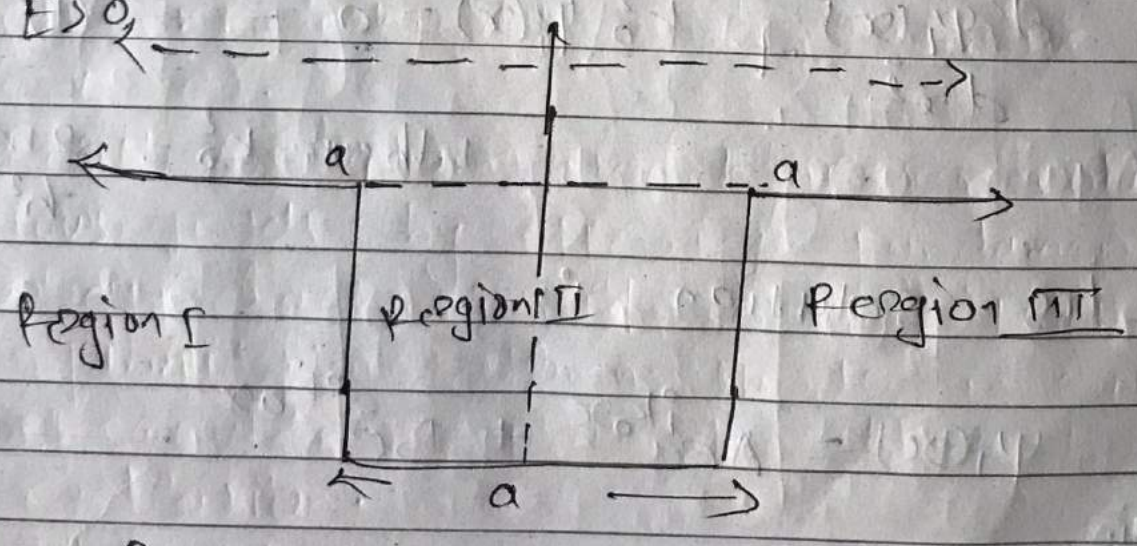


fig - square potential well of finite depth.

The boundry condition of ~~poten~~ square potential is,

$$V(x) = 0 \text{ for } x < 0$$
$$= +|V_0| \text{ for } 0 < x < a$$
$$= 0 \text{ for } x > a$$

Let $\psi_1(x)$ ~~or~~ $\psi_2(x)$ and $\psi_3(x)$ are wavefunction function in region I, II and III. ~~Then~~ Then,

The schrodinger wave eqn in region I

$$\frac{d^2\psi_1(x)}{dx^2} + \frac{2m}{\hbar^2} E\,\psi_1(x) = 0$$

$$\frac{d^2\psi_1(x)}{dx^2} + k_0^2\,\psi_1(x) = 0 \quad \text{—— (1)}$$

Where $k_0^2 = \frac{2mE}{\hbar^2}$.

Its solution is

$$\psi_1(x) = Ae^{ik_0x} + Be^{-ik_0x} \quad \text{—— (2)}$$

Again

The schrodinger wave eqn in region II

$$\frac{d^2\psi_2(x)}{dx^2} + \frac{2m}{\hbar^2}(E + |V_0|)\,\psi_2(x) = 0 \quad \text{—— (3)}$$

Date

Page

$$\frac{d^2\psi_2(x)}{dx^2} + k^2\psi_2(x) = 0 \quad \text{---} (3)$$

$$k^2 = \frac{2m}{\hbar^2}(E + |V_0|)$$

its solution is

$$\psi_2(x) = Ce^{ikx} + De^{-ikx} \quad \text{---} (4)$$

Again,

The schrodinger wave eqn in region III

$$\frac{d^2\psi_3(x)}{dx^2} + \frac{2m}{\hbar^2}E\psi_3(x) = 0$$

$$\frac{d^2\psi_3(x)}{dx^2} + k_0^2\psi_3(x) = 0 \quad \text{---} (5)$$

$$k_0^2 = \frac{2m}{\hbar^2}E$$

its solution is

$$\psi_3(x) = A'e^{ik_0x} + B'e^{-ik_0x}$$

$$\psi_3(x) = A'e^{ik_0x} \quad \text{---} (6)$$

(since $B' = 0$
no reflection
in region III)
no. particle travel from
left to right so
$B'e^{-ik_0x} = 0$)

Date
Page

Using boundry conditon.

$$\psi_1(x)\Big|_{x=0} = \psi_2(x)\Big|_{x=0}$$

$$\psi_1'(x)\Big|_{x=0} = \psi_2'(x)\Big|_{x=0}$$

$$A+B = C+D \quad \text{——— (7)}$$

$$ik_0 A - ik_0 B = ikC - ikD$$

$$A-B = \frac{k}{k_0}(C-D) \quad \text{——— (8)}$$

Adding eqn (7) and (8)

$$2A = \left(1+\frac{k}{k_0}\right)C + \left(1-\frac{k}{k_0}\right)D \quad \text{——— (9)}$$

Again using boundry conditon

$$\psi_2(x)\Big|_{x=a} = \psi_3(x)\Big|_{x=a}$$

$$\psi_2'(x)\Big|_{x=a} = \psi_3(x)\Big|_{x=a}$$

$$Ce^{ika} + De^{-ika} = A'e^{ik_0a} \quad \text{——— (10)}$$

Date
Page

and

$$ikCe^{ika} - ikDe^{-ika} = ik_0A'e^{ik_0a}$$

$$Ce^{ika} - De^{-ika} = \frac{k_0}{k}A'e^{ik_0a} \quad \text{——— (11)}$$

Adding eqn (10) and (11)

$$2Ce^{ika} = \left(1+\frac{k_0}{k}\right)A'e^{ik_0a}$$

$$C = \frac{A'}{2}\left(1+\frac{k_0}{k}\right)e^{i(k_0-k)a} \quad \text{——— (12)}$$

Again,
subtracting eqn (11) from (10)

$$2De^{-ika} = \left(1-\frac{k_0}{k}\right)A'e^{ik_0a}$$

$$D = \frac{A'}{2}\left(1-\frac{k_0}{k}\right)e^{i(k_0+k)a} \quad \text{——— (13)}$$

putting value of C and D in eqn (9)

$$2A = \left[\left\{\left(1+\frac{k}{k_0}\right)\frac{A'}{2}\left(1+\frac{k_0}{k}\right)e^{i(k_0-k)a}\right\} + \left\{\left(1-\frac{k}{k_0}\right)\frac{A'}{2}\left(1-\frac{k_0}{k}\right)e^{i(k_0+k)a}\right\}\right]$$

Date
Page

$$2A = \left[\frac{1}{2}\left(1+\frac{k}{k_0}\right)\left(1+\frac{k_0}{k}\right)e^{-ika} + \left(1-\frac{k}{k_0}\right)\cdot\frac{1}{2}\left(1-\frac{k_0}{k}\right)e^{ika}\right]A'e^{ik_0a}$$

$$2A = \left[\frac{1}{2}\left(2+\frac{k}{k_0}+\frac{k_0}{k}\right)e^{-ika} + \frac{1}{2}\left(2-\frac{k_0}{k}-\frac{k}{k_0}\right)e^{ika}\right]e^{ik_0a}$$

$$2A = \left[\frac{1}{2}\left(2e^{-ika}\right) + \left(\frac{k}{k_0}+\frac{k_0}{k}\right)e^{-ika} + \frac{2e^{ika}}{2} - \left(\frac{k_0}{k}-\frac{k}{k_0}\right)e^{ika}\right]A'e^{ik_0a}$$

$$A = \left[\left(\frac{e^{ika}+e^{-ika}}{2}\right) - \frac{i}{2}\left(\frac{k_0}{k}+\frac{k}{k_0}\right)\left(\frac{e^{ika}+e^{-ika}}{2i}\right)\right]A'e^{ik_0a}$$

$$A = \left[\cos ka - \frac{i}{2}\left(\frac{k_0}{k}+\frac{k}{k_0}\right)\sin ka\right]A'e^{ik_0a}$$

$$\frac{A}{A'} = \left[\cos ka - \frac{i}{2}\left(\frac{k_0}{k}+\frac{k}{k_0}\right)\right]e^{ik_0a}$$

$$\frac{A'}{A} = \frac{1}{\left[\cos ka - \frac{i}{2}\left(\frac{k_0}{k}+\frac{k}{k_0}\right)e^{ik_0a}\right]}$$

2nd part

Transmission cocoefficent (TE)

$$T_E = \left|\frac{A'}{A}\right|^2 = \left(\frac{A'}{A}\right)^* \left(\frac{A}{A}\right)$$

$$T_E = \frac{1}{\left(\cos ka - \frac{i}{2}\left(\frac{k_0}{k} + \frac{k}{k_0}\right)^2 \sin ka\right)\left(\cos ka + \frac{i}{2}\left(\frac{k_0}{k} + \frac{k}{k_0}\right) \sin ka\right)}$$

$$T_E = \frac{1}{\cos^2 ka + \frac{1}{4}\left(\frac{k_0}{k} + \frac{k}{k_0}\right) \sin^2 ka}$$

$$T_E = \frac{1}{1 - \sin^2 ka + \frac{1}{4}\left(\frac{k_0}{k} + \frac{k}{k_0}\right) \sin^2 ka}$$

$$T_E = \frac{1}{1 + \frac{1}{4}\left(\left(\frac{k_0}{k} + \frac{k}{k_0}\right) - 4\right) \sin^2 ka} \quad \text{—— (14)}$$

using $k_0^2 = \frac{2m}{\hbar^2}$ and $k^2 = \frac{2m(E + |V_0|)}{\hbar^2}$ in eqn (14) we got

Date
Page

$$T_E = \frac{1}{1 + \frac{V_0^2 \sin^2 ka}{4E(E+|V_0|)}}$$

Here T_E is maximum when $\sin ka = 0$

$$\sin ka = \sin n\pi$$
$$ka = n\pi$$

$$\sin \sqrt{\frac{2m}{\hbar^2}(E+|V_0|)} = n\pi$$

$$ka = n\pi \qquad \left[ka = \sin\sqrt{\frac{2m}{\hbar^2}(E+|V_0|)}\right.$$

$$a = \frac{n\pi}{k}$$

$$a = \frac{n\cancel{\pi}}{\frac{2\cancel{\pi}}{\lambda}} \qquad k = \frac{2\pi}{\lambda}$$

$$a = \frac{n\lambda}{2}$$

This is the condition of resonance. This shows that the well becomes transparent when width of wall is equal to integral multipal of half of wave ~~the~~ length.

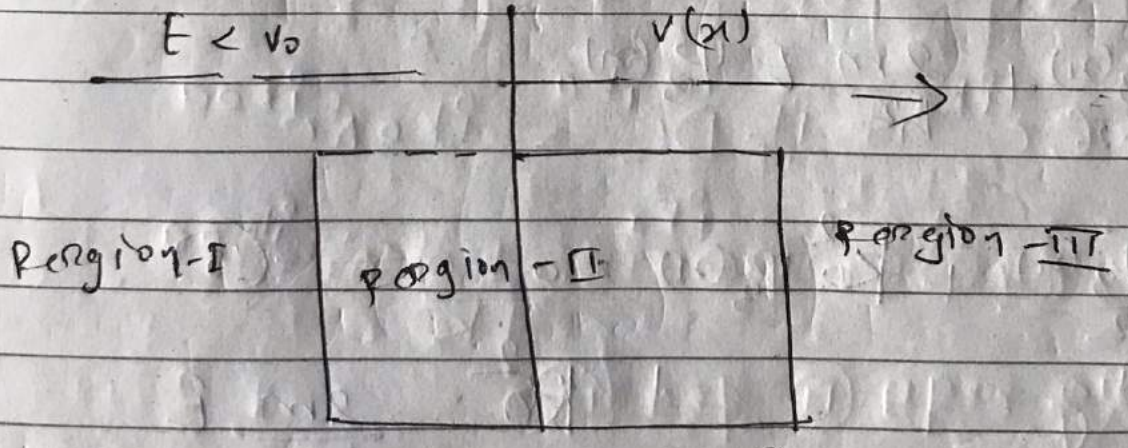
Date
Page

Q6) find the expression for the energy level of a finite square potential well, describe by the potential $V(x)$ for a beam of quantum particles having energy $E < V_0$

$$V(x) = 0 \text{ for } x < 0$$
$$= -V_0 \text{ for } 0 \le x \le 2a$$
$$= 0 \text{ for } x \ge 2a$$

Solution

Lept us consider a potential well as shown as figur. of width 2a. The potential of well V_0 as shown as figur.



The schrodinger wave eqn in region I

$$\frac{d^2\psi_1(x)}{dx^2} + \frac{2m}{\hbar^2} E_1 \psi_1(x) = 0$$

$$\frac{d^2\psi_1(x)}{dx^2} + k_0^2\psi_1(x) = 0 \quad \text{--- (1)}$$

$$k_0^2 = \frac{2m}{\hbar^2}E$$

Schrodinger wave equation in region II

$$\frac{d^2\psi_2(x)}{dx^2} + \frac{2m}{\hbar^2}(E-V)\psi_2(x) = 0$$

$$\frac{d^2\psi_2(x)}{dx^2} + k^2\psi_2(x) = 0 \quad \text{--- (2)}$$

$$k^2 = \frac{2m}{\hbar^2}(E-V)$$

Schrodinger wave eqn in region III

$$\frac{d^2\psi_3(x)}{dx^2} + \frac{2m}{\hbar^2}E\psi_3(x)$$

$$\frac{d^2\psi_3(x)}{dx^2} + k_0^2\psi_3(x) = 0 \quad \text{--- (3)}$$

$$k_0^2 = \frac{2m}{\hbar^2}E$$

The solution of all these eqn are

$$\psi_1(x) = Ae^{ik_0x} + Be^{-ik_0x} \quad \text{--- (4)}$$

$$\psi_2(x) = Ce^{ikx} + De^{-ikx} \quad \text{--- (5)}$$

$$\psi_3(x) = A' e^{ik_0x} \quad \text{--- (6)}$$

No, wave reflected in region III so,
$B' = 0$

Applying boundry conditions:

$$\psi_1(x)\Big|_{x=-a} = \psi_2(x)\Big|_{x=+a}$$

$$\psi_1'(x)\Big|_{x=-a} = \psi_2'(x)\Big|_{x=+a}$$

eqn (4) and (5) becomes

$$Ae^{-ik_0a} + Be^{ik_0a} = Ce^{ika} + De^{-ika} \quad \text{--- (7)}$$

and,

$$ik_0Ae^{-ika} - ik_0Be^{ik_0a} = ikCe^{-ika} - ikDe^{ika}$$

$$Ae^{-ik_0a} - Be^{ik_0a} = \frac{k}{k_0} Ce^{-ika} - De^{ika} \quad \text{--- (8)}$$

Adding eqn (7) and (8)

$$2Ae^{-ik_0a} = \left(1 + \frac{k}{k_0}\right) Ce^{-ika} + \left(1 - \frac{k}{k_0}\right) De^{ika}$$

$$A = \frac{C}{2}\left(1 + \frac{k}{k_0}\right) e^{-ika} \cdot e^{ik_0a} + \frac{D}{2}\left(1 - \frac{k}{k_0}\right) e^{ika \cdot ik_0a} \quad \text{--- (9)}$$

Similarly equation 8 from (7) subtract we get

$$B = \frac{C}{2}\left(1 - \frac{k}{k_0}\right)e^{-ika} \cdot e^{ik_0a} + \frac{D}{2}\left(1 + \frac{k}{k_0}\right)e^{ika} \cdot e^{-ik_0a} \quad \text{--- (10)}$$

Again using boundry condition,

$$\psi_2(x)\Big|_{x=a} = \psi_3(x)\Big|_{x=a}$$

$$\psi_2'(x)\Big|_{x=a} = \psi_3'(x)\Big|_{x=a}$$

Then eqn (5) and (6) becomes

$$Ce^{ika} + De^{-ika} = A'e^{ik_0a} \quad \text{--- (11)}$$

$$ikCe^{ika} - ikDe^{-ika} = ik_0A'e^{ik_0a}$$

$$Ce^{ika} - De^{-ika} = \frac{k_0}{k}A'e^{ik_0a} \quad \text{--- (12)}$$

Adding eqn (11) and (12)

$$2Ce^{ika} = \left(1 + \frac{k_0}{k}\right)A'e^{ik_0a}.$$

$$C = \frac{A'}{2}\left(1 + \frac{k_0}{k}\right)e^{ik_0a} \cdot e^{-ika} \quad \text{--- (13)}$$

Subtracting 12 from (11) we get

$$D = \frac{A'}{2}\left(1 - \frac{k_0}{k}\right) e^{ik_0a} \cdot e^{ika} \quad \text{—— (14)}$$

Using value of C and D in eqn (9) we get

$$A = \frac{A'}{4}\left(1 + \frac{k}{k_0}\right)\left(1 + \frac{k_0}{k}\right) e^{2ik_0a} \cdot e^{-2ika} +$$

$$\frac{A'}{4}\left(1 - \frac{k}{k_0}\right)\left(1 - \frac{k_0}{k}\right) e^{2ik_0a} \cdot e^{2ika}$$

$$A = \frac{A'}{4} e^{2ik_0a}\left[\frac{(k_0+k)^2 e^{-2ika} - (k-k_0)^2 e^{2ika}}{k_0k}\right]$$

$$\frac{A}{A'} = \frac{\left[(k_0+k)^2 e^{-2ika} - (k-k_0)^2 e^{2ika}\right]}{4k_0k} e^{2ik_0a}$$

$$\frac{A'}{A} = \frac{4k_0k\, e^{-2ik_0a}}{(k_0^2+k^2)\left(e^{-2ika} - e^{2ika}\right) + 2k_0k\left(e^{-2ika} + e^{2ika}\right)}$$

$$\frac{A'}{A} = \frac{4k_0k\, e^{-2ik_0a}}{2i(k_0^2+k^2)\left(\frac{e^{-2ika} - e^{2ika}}{2i}\right) + 4k_0k\left(\frac{e^{2ika} + e^{-2ika}}{2}\right)}$$

A'

$$\boxed{\frac{A'}{A} = \frac{4k_0k\,e^{2ik_0a}}{2i(k_0^2+k^2)}}$$

$$\frac{A'}{A} = \frac{4k_0k\,e^{2ik_0a}}{-2i(k_0^2+k^2)\sin 2ka + 4k_0k\cos 2ka}$$

$$\frac{A'}{A} = \frac{4k_0k\,e^{2ik_0a}}{4k_0k\cos 2k - 2i(k_0^2+k^2)\sin 2ka} \quad \longleftarrow (15)$$

Then the transmission coefficient is given by.

$$T = \left|\frac{A'}{A}\right|^2 = \left(\frac{A'}{A}\right)^* \times \left(\frac{A'}{A}\right)$$

$$T = \frac{(4k_0k\,e^{-ik_0a})^2}{(4k_0k\cos 2ka - 2i(k_0^2+k^2)\sin 2ka)^2}$$

$$= \frac{16k_0^2k^2}{16k_0^2k^2\cos^2 ka + 4(k_0^2+k^2)^2\sin^2 2ka}$$

$$T = \frac{16k_0^2k^2}{16k_0^2k^2\cos^2 2ka + 4(k_0^2 \cancel{+} k^2)\sin^2 2ka}$$

$$T = \frac{1}{\cos^2 2ka + \frac{(k_0^2+k^2)}{4k_0^2k^2}\sin^2 ka}$$

$[\cos^2 a = 1-\sin^2 a]$

$$T = \frac{1}{1-\sin^2 2ka + \left(\frac{(k_0^2+k^2)^2}{4k_0^2k^2}\right)\sin^2 ka}$$

$$T = \frac{1}{1+\left[\sin^2 2ka\left(\frac{(k_0^2+k^2)^2}{4k_0^2k^2}-1\right)\right]}$$

$$T_E = \frac{1}{1+\left(\frac{k_0^2-k^2}{2k_0k}\right)^2\sin^2 2ka} \quad \text{—— (16)}$$

$$k_0^2 = \frac{2mE}{\hbar^2} \qquad k^2 = \frac{2m}{\hbar^2}(E+|V_0|)$$

so

$$\frac{k^2-k^2}{2k_0k} = \frac{\frac{2mE}{\hbar^2} - \frac{2m}{\hbar^2}(E+|V_0|}{2\sqrt{\frac{2mE}{\hbar^2}}\cdot\sqrt{\frac{2m(E+|V_0|)}{\hbar^2}}}$$

$$\left(\frac{k_0^2-k^2}{2k_0k}\right)^2 = \frac{-|V_0|}{2\sqrt{E}\cdot\sqrt{E+|V_0|}}$$

$$\left(\frac{k_0^2-k^2}{2k_0k}\right)^2 = \frac{V_0^2}{4E(E+|V_0|)}$$

eqn (16) becomes

$$T_E = \frac{1}{1 + \frac{v_0^2}{4E(E+|v_0|)\sin^2 2ka}}$$

$$T_E = \frac{4E(E+|v_0|)}{4E(E+|v_0| + v_0^2 \sin^2 2ka}$$

This gives transmission coefficient of the particle in potential well of finite depth. when $E < v_0$

Again reflection coefficient

$$R_E = 1 - T_E$$

$$R_E = \frac{V_0^2 \sin 2ka}{4E(E+|V_0|) + v_0^2 \sin^2 2ka}$$

Which is required reflection coefficient

Date
Page

Q. 2 OR What do you mean by cold emission of electron.? Find transmission probability following electron from the surface of the metal due to the electric field. Discuss it in the contex of scaning tunnelling miscroscope (STM).

Soln:

1st part

Cold emission refers to the emission of electron from a surface without need for heating the materials. It can occour througy various mechanism such as field emission, where as electrons are emitted due to strong electric field.

2nd Part

A scanning tunneling miscroscope (STM) is an instrument for producing image of a surface atomic level. It was developed in 1982 by Gerd Binning and Heinrich Rohrer. For on STM. good resolution is considered to be 0.1nm lateral resolution and 0.1 nm depth resolution. The STM can be used not only in Ultra high vacum but ~~also~~ also in air ranging from near 0K to few hundred Celsius.

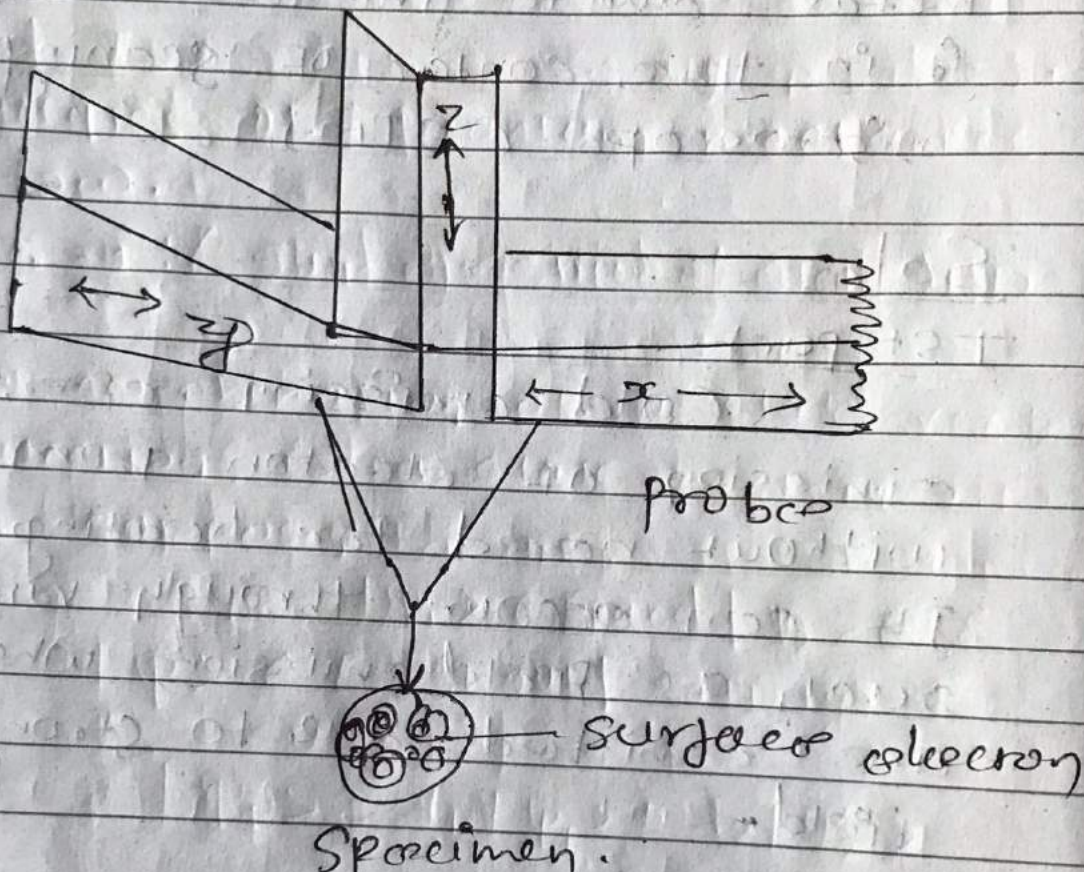


Fig - schematic diagram of (STM)

The STM based on the concept of quantum tunneling when a conducting ~~tip~~ tip is brought very near to the surface to be examined, a ~~be~~ bias applied between the two can allow electrons to tunnel through the vacuum between them. The resulting tunneling current is a function of tip position, applied voltage, local density of state of the sample. Information is acquired by monitoring the current as tips position scans across the surface, and it usally displayed in the image clean and stable surfaces, sharp tips, excellent vibration control and sophisticated electrons.

Date
Page

Q.3 Discuss postulates of quantum mechanics How these postulates demand conservation of probability? explain.

Solution

postulates of quantum mechanics are as below.

Postulate -1 : (Superposition of state)

A Linear combination of a state vector wave function is a state vector.
Let us suppose,

$$|\psi_1\rangle \in E \text{ and } |\psi_2\rangle \in E$$

Then, $|\psi\rangle \in E$ should satisfy.

$$|\psi_1\rangle = \alpha_1 |\psi_1\rangle + \alpha_2 |\psi_2\rangle$$

This relation implies superposition principle in quantum mechanics. It is based upon linear combination of state.

Postulate-2 :- (Operator (physical quantity)

Every measurable physical quantity (A) can be ~~operator~~ expressed by an operator $\hat{A}$ acting on the state space E.

Date
Page

physical operator $\longleftrightarrow$ operator $\longleftrightarrow$ basic set.

explanation,

All eigen vectors of an operator $\hat{A}$ acting in the state space ϵ constitute a basic set $\hat{A}|u_i\rangle = q_i|u_i\rangle$ Here, $|u_i\rangle$ represent basic set and q_i represent the eigen value of eigen vector. The basic set can be expressed in term of wave function ψ.

$$\therefore |\psi\rangle = \sum_i c_i|u_i\rangle$$

Postulate - 3 :- Eigen value | Eigen vectors

The nature of quantum mechanics in measurment of physical quantity A is always real. The real value lies in one of the eigen value of the eigen vector. This eigen vector must be Hermition in nature.

$$\hat{A}|u_i = q_i|u_i\rangle$$

where $|u_i\rangle$ is basic set and q_i is real number and $\hat{A}$ is hermition operator.

Date

Page

$\hat{A}|u_i\rangle = q_i|u_i\rangle$ for discrete system.

$\hat{A}|u_\alpha\rangle = q_\alpha|u_\alpha\rangle$ for contineous system

ii) q_i and q_α represent eigen values.

<u>postulate-4</u> <u>Probability</u> and normalization

When a physical quantity A is measured on a physical system of normalization state ψ. The probability of obtaning non-degenerate value of an $\hat{A}$ is,

$$P = |\langle u_A|\psi\rangle|^2 = |c_n|^2$$
$$P = (\psi^* \psi \, dv)$$

Where $\langle u_n|\psi\rangle$ is expansion cofficent.

If the measurment of physical quantity A on physical system on normalized state ψ gives the result a_n. The state of the system immediately after mesurment is normalized projection

$$\frac{P_n|\psi\rangle}{\sqrt{\langle\psi|P_n\rangle\psi}}$$ to the sub space associated with a_n.

Date
Page

Postulate - 5 :- ~~Time~~ Time evolution of the System of schrodinger wave equation

This concept gives the time variation of state function and evolution of this function in latter time when same physical observable operator on that state. This postulate is the differential eqn of first order with respect to time.

i.e

$$H|\psi\rangle = \frac{i\hbar\, \partial|\psi\rangle}{\partial t}$$

This, is also called time dependent schrodinger equation.

Postulate - 6 :-

An electron possess an intrinsic angular momentum called spin vector with component S_x, S_y and S_z is being a two valued observable with possible eigen value $\pm \hbar/2$

for each $\vec{S} = \hat{e}_1 \cdot S_x + \hat{e}_2 S_y + \hat{e}_3 S_z$

$$\therefore \vec{S} = \hat{e}_1 S_x + \hat{e}_2 S_y + \hat{e}_3 S_z$$

Date ____
Page ____

2nd part

The probability of all possible outcomes of a mesurment must sum to 1. This conservation of probability ensures that the probabilities are normalized and that the ~~proba~~ out-come of a measurment is certain to be one of the possible eigen values. Without conservation of probabi-lity, the predictions of quantum mechanics would not match experimen results, and the theory wold not be consistent. Therefore it is a fundamenta requirement for the validity of quantu mechanics.

Solve by
Aashish Chand

Date
Page

Q.4 Find the expression of the spherical harmonics when $l=2$ and $m=\pm 1$ in terms of spherical polar cordinates (θ and ϕ). Make plot of the above spherical harmonics verses θ in any plane through z-axis.

Solution

Here, the spherical harmonics

$$Y_l^m(\theta,\phi) = \epsilon\sqrt{\frac{(2l+1)(l-m)!}{4\pi(l+m)!}}\, e^{im\phi} P_l^m(\cos\theta)$$

where $\epsilon = (-1)^m$ for $m \geq 0$ and $\epsilon = 1$ for $m \leq 0$

We have,

$$\int_0^{2\pi}\int_0^{\pi}\left[Y_l^m(\theta,\phi)\right]^*\left[Y_{l'}^{m'}(\theta,\phi)\right]\sin\theta\, d\theta\, d\phi$$

$$= \delta_{ll'}\delta_{mm'}$$

Here, $l=2$ and $m = +1, -1$

$$Y_2^{\pm 1} = \mp\left(\frac{15}{8\pi}\right)^{1/2}\sin\theta\, e^{\pm i\phi}$$

Also,

$$Y_0^0 = \pm\frac{1}{\sqrt{4\pi}}$$

Date
Page

Some few spherical harmonics are

$$Y(0,0) = \frac{e^{\pm i\cdot 0\cdot\phi}}{\sqrt{2\pi}} \sqrt{\frac{(2\cdot 0+1)(0-0)!}{2(0+0)!}}\, P_0^0(\cos\theta) = \frac{1}{(4\pi)^{1/2}}$$

$$Y(1,0) = \frac{e^{\pm i\cdot 1\cdot\phi}}{\sqrt{2\pi}} \sqrt{\frac{(2\cdot 1+1)(1-0)!}{2(1+0)!}}\, P_1^0(\cos\theta)$$

$$= \left(\frac{3}{4\pi}\right)^{1/2} \cos\theta$$

$$Y(1,\pm 1) = \frac{e^{\pm i\cdot 1\cdot\phi}}{\sqrt{2\pi}} \sqrt{\frac{(2\cdot 1+1)(1-\pm 1)!}{2(1\pm 1)!}}\, P_1^{\pm 1}(\cos\theta)$$

$$= 8\left(\frac{3}{8\pi}\right)^{1/2} \sin\theta\, e^{\pm i\phi}$$

$$Y(2,0) = \frac{e^{\pm i\cdot 0\cdot\phi}}{\sqrt{2\pi}} \sqrt{\frac{2(2\cdot 2+1)(2-0)!}{2(2+0)!}}\, P_2^0(\cos\theta)$$

$$= \left(\frac{5}{16\pi}\right)^{1/2} (3\cos^2\theta - 1)$$

$$Y(2,2) = \frac{e^{\pm i\cdot 2\cdot\phi}}{\sqrt{2\pi}} \sqrt{\frac{(2\cdot 2+1)(2-2)!}{(2(2+2)!}}\, P_2^2(\cos\theta)$$

$$= \left(\frac{15}{32\pi}\right)^{1/2} \sin^2\theta\, e^{\pm 2i\phi}$$

Date
Page

Q(5) Discuss free particle radial function and show that the energy levels of particles are discrete when placed in a spherical box — 8 marks

Soln
Consider a particle of mass m with potential V(r). Then, the schrodinger wave equation of cartesian cordinates system in the three dimension is given by:

$$\nabla^2 \psi + \frac{2m}{\hbar^2}[E - V(r)]\psi = 0 \quad \text{— (1)}$$

for free particle potential $V(r) = 0$

$$\nabla^2 \psi + \frac{2m}{\hbar^2} E \psi = 0$$

$$\text{or } \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} + \frac{2m}{\hbar^2} E \psi = 0 \quad \text{— (2)}$$

To solve eqn (1), let us separate wave function ψ as,

$$\psi = \psi(x, y, z) = X(x)\, Y(y)\, Z(z) \quad \text{— (3)}$$

using eqn (3) in eqn (2), we got.

Date
Page

$$YZ\frac{\partial^2 X}{\partial x^2} + XZ\frac{\partial^2 Y}{\partial y^2} + XY\frac{\partial^2 Z}{\partial z^2} + \frac{2m}{\hbar^2}EXYZ = \quad (4)$$

Dividing eqn (4) by XYZ

$$\frac{1}{X}\frac{\partial^2 X}{\partial x^2} + \frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} + \frac{1}{Z}\frac{\partial^2 Z}{\partial z^2} + \frac{2m}{\hbar^2}E = 0$$

$$\frac{1}{X}\frac{\partial^2 X}{\partial x^2} = -\frac{2mE}{\hbar^2} - \frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} - \frac{1}{Z}\frac{\partial^2 Z}{\partial z^2} = (-k_x^2) \text{ say}$$

$$\therefore \frac{\partial^2 X}{\partial x^2} + k_x^2 X = 0$$

Similarly

$$\frac{\partial^2 Y}{\partial y^2} + k_y^2 X = 0$$

$$\frac{\partial^2 Z}{\partial z^2} + k_z^2 Z = 0$$

(5)

Where $k^2 = k_x^2 + k_y^2 + k_z^2 = \frac{2mE}{\hbar^2} = \frac{p^2}{\hbar^2}$

The normalized plane wave solution for equation (2) is

$$\psi = u_k(\vec{r}) = \frac{1}{(2\pi)^{3/2}} e^{i(\vec{k}\cdot\vec{r})}$$

~~Some~~

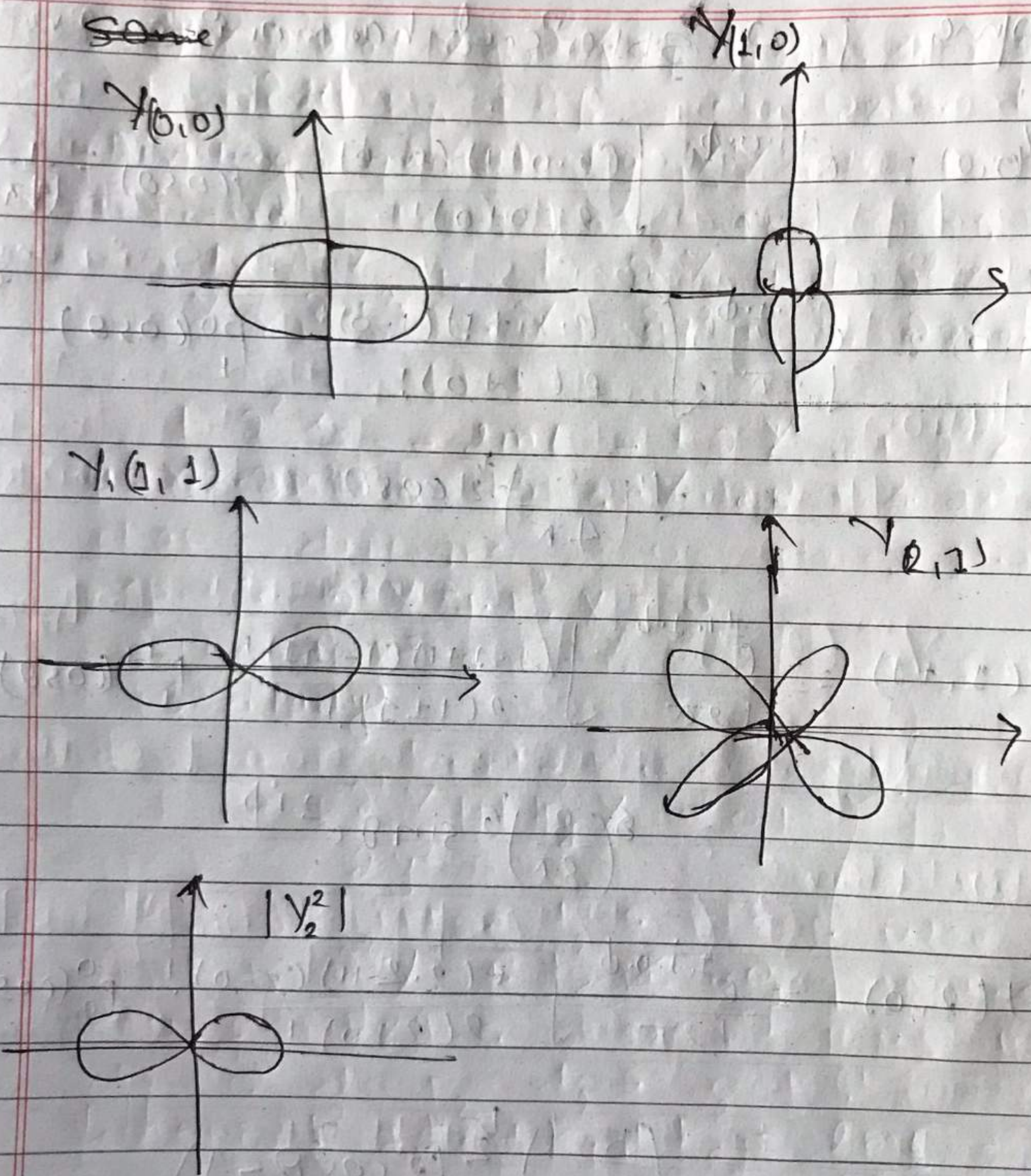


Fig: polar graphs of probability $|Y_m^l|$ vs θ in any plane through z-axis for $l = 0, 1, 2, \ldots n$.

Date
Page

This solution is Eigen function of the three components of the momentum $\vec{P}$. The momentum Operators.

$$\hat{P} = -i\hbar\nabla = -i\hbar\left[\hat{e}_x\frac{\partial}{\partial x} + \hat{e}_y\frac{\partial}{\partial y} + \hat{e}_z\frac{\partial}{\partial z}\right]$$

$\hat{e}$ is unit vectors.

Now we take,

$$\hat{P}u_k(\vec{r}) = -i\hbar\left[\hat{e}_x\frac{\partial}{\partial x} + \hat{e}_y\frac{\partial}{\partial y} + \hat{e}_z\frac{\partial}{\partial z}\right] \times \frac{1}{(2\pi)^{3/2}} e^{\frac{i}{\hbar}(P_x x + P_y y + P_z z)}$$

$$\hat{P}u_k(\vec{r}) = \frac{-i\hbar}{(2\pi)^{3/2}} \cdot \frac{i}{\hbar}\left[\hat{e}_x P_x + \hat{e}_y P_y + \hat{e}_z P_z\right] e^{\frac{i}{\hbar}(P_x x + P_y y + P_z z)}$$

$$\hat{P}u_k(\vec{r}) = \frac{\vec{P}}{(2\pi)^{3/2}} \cdot e^{i/\hbar(\vec{P}\cdot\vec{r})}$$

$$\hat{P}u_k(\vec{r}) = \hat{P}u_k(\vec{r})$$

Where $u_k(\vec{r}) = \frac{1}{(2\pi)^{3/2}} e^{i(\vec{k}\cdot\vec{r})}$

Date

Page

A plane wave perpendicular to $\vec{k}$ is defined by $\vec{k}\cdot\vec{r}$ = constant. The projection of every point of this infinite plane has the same value

$$r_{\parallel} = \frac{(\vec{k}\cdot\vec{r})}{|\vec{k}|} \text{ is in direction of } \vec{k}.$$

u_k has the same value for the phase difference 2π on planes $\vec{k}\cdot\vec{r}$ = constant $+ 2\pi$.

The distance between two successive planes is

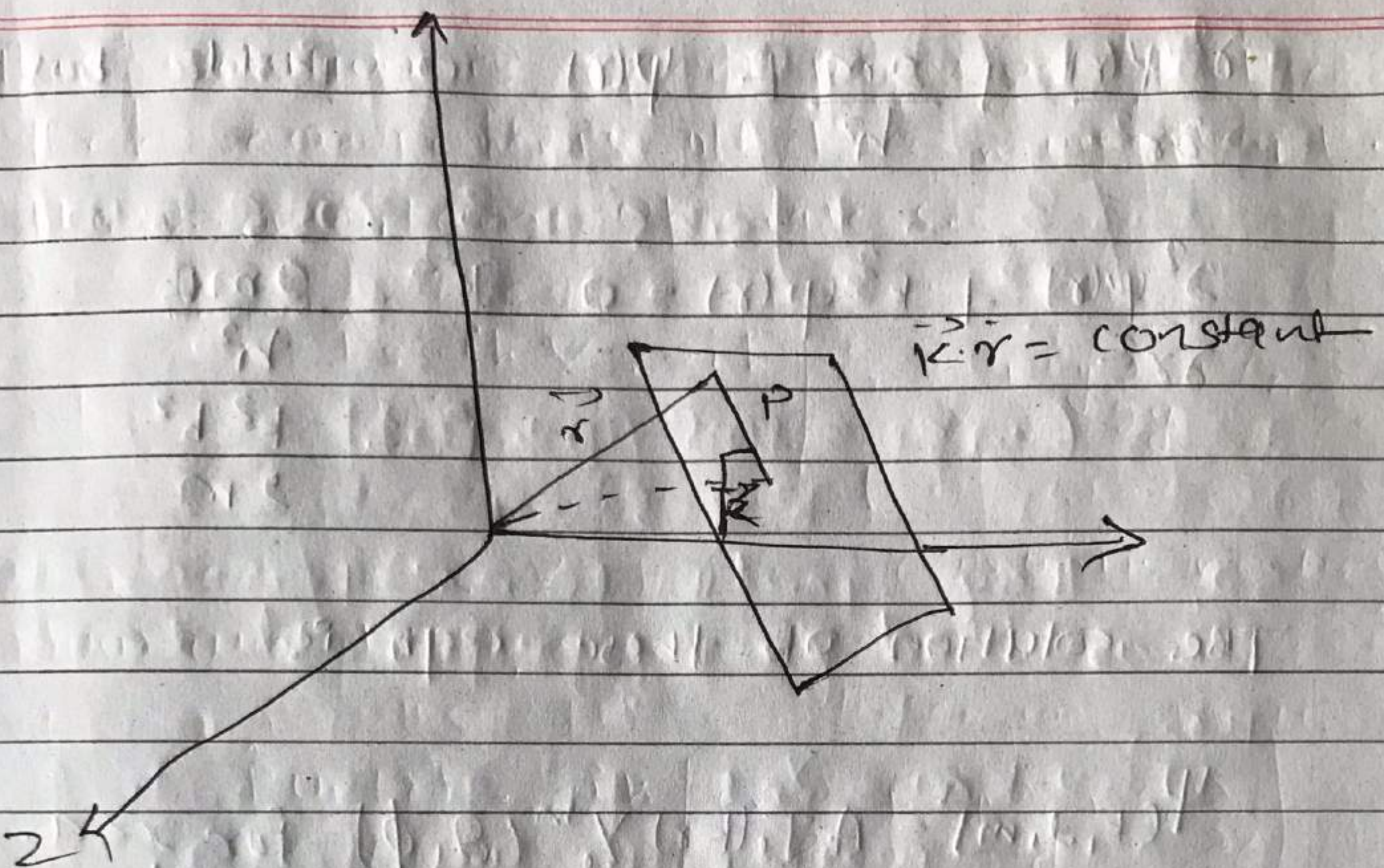
$$\frac{k\cdot\vec{r}' - \vec{k}\cdot\vec{r}}{|\vec{k}|} = \frac{2\pi}{|\vec{k}|} = d.$$

This justifies the plane waves which are of infinitely extent and occupy all the space. Thus the time dependent radial wave function is given by

$$\psi_k(\vec{r},t) = u_k(\vec{r})\, e^{-\frac{iEt}{\hbar}}$$

$$\therefore \psi_k(\vec{r},t) = e^{i/\hbar\,(\vec{p}\cdot\vec{r} - Et)}$$

Date
Page



<u>2nd part</u> particle in a spherical box.

Let us consider a spherical potential box of potential is given by

$$V(r) = 0 \text{ for } r < a$$
$$\infty \text{ for } r \geq a$$

The schrodinger wave eqn inside the box is, given by.

$$\frac{\partial^2 \psi(r)}{\partial x^2} + \frac{2m}{\hbar^2}(E - V(r))\psi(r) = 0$$

$$\frac{\partial^2 \psi(r)}{\partial r^2} + \frac{2m}{\hbar^2} E \, \psi(r) = 0$$ inside box $V=0$

$$\frac{\partial^2 \psi(r)}{\partial r^2} + k^2 \psi(r) = 0 \quad k^2 = \frac{2mE}{\hbar^2}$$

$$E = \frac{k^2 \hbar^2}{2m}$$

The solution of these eqn is

$$\psi_{(k,l,m)} = A J_l(kr) Y_l^m(\theta, \phi) \text{ for } x < a.$$

Where A is amplitude of wave $J_l(kr)$ is Bessels function and $Y_l^m(\theta,\phi)$ is Spherical harmonics.

As, $V(r \geq a) = \infty$;

The boundry condition are $J_l(ka) = 0$

$$J_{l+\frac{1}{2}}(ka) = 0$$

we know,

$$J_0 = \frac{\sin \rho}{\rho}$$

Date
Page

$$J_1 = \frac{\sin\rho}{\rho^2} - \frac{\cos\rho}{\rho^2}$$

$$J_2 = \left(\frac{3}{\rho^3} - \frac{1}{\rho}\right)\sin\rho - \frac{3}{\rho^3}\cos\rho$$

Where $\rho = kr$

Now, As; $J_0(\rho) = 0$,

$$\frac{\sin\rho}{\rho} = 0 \Rightarrow \sin\rho = 0$$

$$\sin\rho = \sin n\pi$$

$$\rho = n\pi$$

Again $J_1(\rho) = 0$

$$\frac{\sin\rho}{\rho^2} - \frac{\cos\rho}{\rho} = 0$$

$$\frac{\sin\rho}{\cos\rho} = \frac{\rho^2}{\rho}$$

$$\tan\rho = \rho$$

Again,

$$J_2(\rho) = 0$$

$$\left(\frac{3}{\rho^3} - \frac{1}{\rho}\right)\sin\rho - \frac{3}{\rho^2}\cos\rho = 0$$

$$\tan\rho = \frac{3\rho}{3-\rho^2}$$

So, they tends to $n\pi$ for even l ~~or~~ and $\left(n+\frac{1}{2}\right)\pi$ for odd l.

Then,

$$ka = X_{nl}$$

$$k = \frac{X_{nl}}{a}$$

Now, energy,

$$E = \frac{\hbar^2 k^2}{2m}$$

$$\boxed{E = \frac{\hbar^2 X_{nl}^2}{2ma^2}}$$

Date
Page

Q(6) Seprate radial part from the schrodi-
-nger equation using the methods of Sepration of variables and find the solution of Hydrogen atoms.

Solution

The schrodinger's equation in spherical polar co-ordinates for the problems of hydrogen atom is that in the from it may be seprated into three independent equations, each involving only a single co-ordinate.

Let us first seprate the wave function $\psi(r, \theta, \phi)$ into radial and angular parts.

Here,

$$\psi(r, \theta, \phi) = R(r)\, \psi(\theta, \phi) \quad --- (1)$$

In which $R(r)$ is independent of the angles and $Y(\theta, \phi)$ is independent for r

The schrodinger's equation is

$$\frac{\partial}{r^2 \partial r}\left(r^2 \frac{\partial \psi}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial \psi}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 \psi}{\partial \psi^2}$$

$$+ \frac{2\mu}{\hbar^2}\left[E - V(r)\right] RY = 0$$

Deviding RY and multiplying by r^2 on both sides,

Date
Page

$$\frac{1}{R}\frac{\partial}{\partial r}\left(r^2\frac{\partial R}{\partial r}\right) + \frac{1}{Y}\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial Y}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2 Y}{\partial\varphi^2}\right] + \frac{2\mu r^2}{\hbar^2}[E - V(r)] =$$

$$\frac{1}{R}\frac{\partial}{\partial r}\left(r^2\frac{\partial R}{\partial r}\right) + \frac{2\mu r^2}{\hbar^2}[E - V(r)] = -\frac{1}{Y}\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial Y}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial}\right.$$

$= d$ (say)

Thus above eqn gives radial equation of the form.

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial R}{\partial r}\right) + \left\{\frac{2\mu}{\hbar^2}[E - V] - \frac{d}{r^2}\right\}R = 0$$

Date
Page

Q(7) Attempt All question.

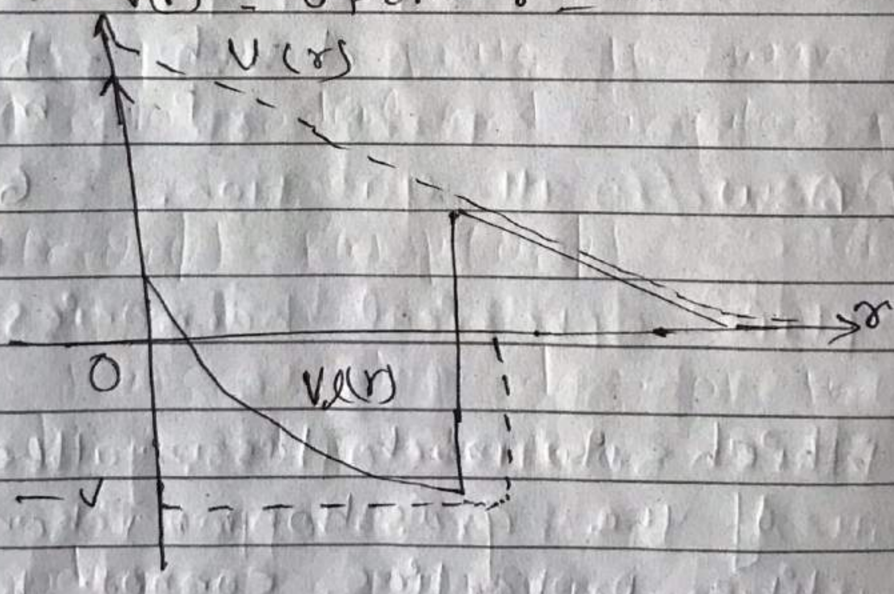
(a) construct spherical potential well and discuss behaviour of free particle wave function in it.

Solution

Consider a square potential well as shows in fig. for which the potential. function is definced as,

$$V(r) = -V_0 \text{ for } r \leq a$$

$$V(r) = 0 \text{ for } r \geq a$$



partical confined with in a spherical region of space where the potential energy is zero but outside of this region, the potential energy becomes infinite.

Date
Page

OR (b) Find De-Broglie wavelength of 10 gm grass-hopper jumping at the speed of 80km/hr in a straight field.

Solution

Given,

mass (m) = 10g

$= 10\times10^{-3}$ kg

velocity (v) = 80 km/hr

wavelength (λ) = ?

We have de-Broglie wavelength.

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

$$= \frac{6.62\times10^{-34}}{10\times10^{-3}\times80}$$

$$= 8.275\times10^{-32} m.$$

Which is much lesser then the dimension of the grasshopper, hence, wave like properties cannot be observed.

Date
Page

Q) Classical physics could not explain the behaviour of a black body radiator at very short wavelength. What was this problem? explain.

Solution

Classical physics failed to explain the behaviour of a black body radiator at very short wavelength due to uv-catastrophe problems. According to classical physics, the intensity of radiations should increase infinitely as the wavelength decreases, leading to an absurd prediction that object would emit infinite amount of energy at short wavelength. This discrepancy between theory and observation was a major challange that classical physics couldn't reconcile.

Q7b) Discuss:

Date
Page

Q(7)(b) Describe the meaning of parity in quantum mechanics.

Solution

The parity refers to the behaviour of wave function under space inversion. The space inversion refer to the behaviour of reflection of spatial (x, y, z) cordinate about the orgion. let we take wave function $\psi(x)$, and the parity operator is defined as,

$$\hat{\pi}\psi(x) = \psi(-x) \text{ and } \hat{\pi}\psi(-x) = \psi(x).$$

Q(8) Answer all question.

(a) explain spherical harmonics.

Soln

The solution of angular equation is called spherical harmonics. The angular part

$Y_{(lm)}(\theta, \phi)$ of the complete wave function, which is the solution

$$\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2 y}{\partial\phi^2} + \alpha y = 0$$

Where $\alpha = l(l+1)$ is called spherical harm

Date
Page

harmonics.

we have $Y_{lm}(\theta, \phi) = \Phi(\theta)\,\Phi(\theta) = \frac{1}{\sqrt{2\pi}} e^{Iim\phi}$

$\sqrt{\frac{(2l+1)(l-m)!}{2(l+m)!}} P_l^m$

By Aashish-chand

B.S.C 4th year 2080

harmonics.

we have,

$$Y_{lm}(\theta, \phi) = \Theta(\theta)\,\Phi(\phi) = \frac{1}{\sqrt{2\pi}} e^{\pm im\phi} \sqrt{\frac{2(l+1)(l-m)!}{2(l+m)!}}$$

$$\cdot P_l^m \cos\theta$$

$$= \frac{\epsilon}{\sqrt{2\pi}} \sqrt{\frac{(2l+1)(l-m)!}{2(l+m)!}}\, P_l^m(\cos\theta) e^{im\phi}$$

where, $\epsilon = (-1)^m$ for $m > 0$ $\epsilon = 1$ for $m \geq 0$

Few spherical harmonics are

$$Y(0,0) = \frac{1}{(4\pi)^{1/2}}$$

$$Y(1,0) = \left(\frac{3}{4\pi}\right)^{1/2} \cos\theta$$

$$Y(1 \pm 1) = \left(\frac{3}{8\pi}\right)^{1/2} \sin\theta\, e^{\pm i\phi}$$

$$Y(2,0) = \left(\frac{5}{16\pi}\right)^{1/2} (3\cos^2\theta - 1)$$

Ans

Date
Page

Q(b) What do you mean by the orthogonality ? explain

Solution

If the scalar product of a function $\psi_1(x)$ and the complex conjugate $\psi_2^*(x)$ of a function $\psi_2(x)$ vanishes when integrated with respect to x over the ~~ent~~ entire rang of x the function $\psi_1(x)$ and $\psi_2(x)$ are said to be mutually ortogonal.

$$\text{i.e.} \int \psi_2(x)\,\psi_1(x)\,dx = 0$$

Where all variables are denoted by a single letter x. This is known as orthogonality.

8.(C) What is rigid rotor ? Discuss

Soln.

The system rotates about an axis passing through center of mass and normal to the plane contining two masses m_1 and m_2. The system is called rigid rotator. because there is no radial motion.

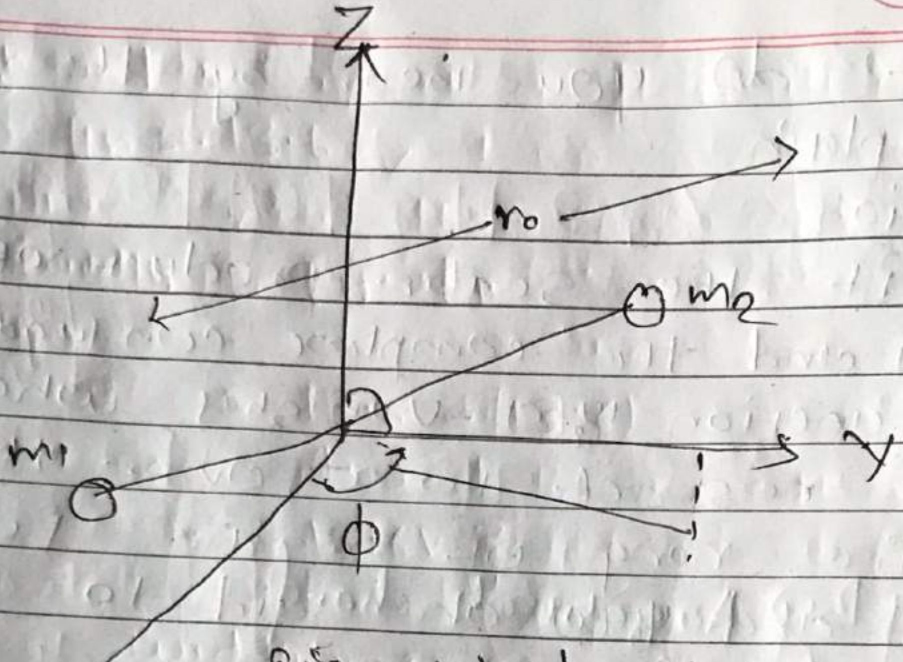


fig - rigid rotator.

Q8(c) Discuss meaning of group velocity.

Solution

The group velocity is velocity of transferences of the energy of the wave product from one point to the another point point the velocity of the group of wave or wave product is equal to the mechanical velocity of the particle is called group velocity.

Mathematically, $v_g = \frac{d\omega}{dK}$.

Date
Page

Q(a) An electron in the n=2 state of hydrogen remain there on the average of about 10^{-8} s before making transition to n=1 state.

(a) Estimate the uncertainity in the energy of the n = 2 state.

(b) What function of transition energy is this? What is the width of this line in the separation of hydrogen atom.

Soln Given,

state (n) = 2.

Time (T) = 10^{-8} sec.

State (n) = 1

(h) = 6.63×10^{-34} kg

(e) = 1.6×10^{-19} J/eV

(a) Soln

$\Delta E \cdot T \geq h$

$$\Delta E = \frac{h}{T} = \frac{1.055 \times 10^{-34}}{10^{-8}}$$

$$= 1.055 \times 10^{-26} J$$

$$\Delta E \geq \frac{1.05 \times 10^{-26}}{1.6 \times 10^{-19}} = 6.59 \times 10^{-8} eV$$

Date
Page

(b) soln

$$E_n = \frac{13.6}{n^2} eV$$

width $= E_n - E_1$.

$$= \frac{13.6}{4} - (13.6)$$

$$= 13.6 \times \frac{3}{4}$$

width $= 10.2 eV$

Date
Page

Q(11) If A and B are two Hermitian operator. Show that AB and BA may not be Hermitian but that (AB + BA) and (AB - BA) are both Hermitian.

Solution

(i) Since A and B are Hermitian, we have,

$$\int \psi_m^* A \psi_n dx = \int A^* \psi_m^* \psi_n dx.$$

$$\int \psi_m^* B \psi_n dx = \int B^* \psi_m^* \psi_n dx$$

$$\int \psi_m^* (AB - BA) \psi_n dx = \int \psi_m^* AB \psi_n dx + \int \psi_m^* BA \psi_n dx$$

$$= \int B^* A^* \psi_m^* \psi_n dx + \int A^* B^* \psi_m^* \psi_n dx$$

$$= \int (AB + BA)^* \psi_m^* \psi_n dx$$

Hence, AB + BA is Hermititan.

(ii) $\int \psi_m^* (AB - BA)$ is not - Hermitian

Aashish

Date
Page

(12) for a one-dimensional wave function of the form $\psi(x,t) = A\sin[i\phi(x,t)]$. find the probability current density.

Soln →

Here, given

$\psi(x,t) = A\sin[i\phi(x,t)]$

$\psi^* = A\cos[x,t)\cdot t$

probability current density.

$$S = \frac{\hbar}{2mi}\left[\psi^*\frac{\partial\psi}{\partial x} - \psi\frac{\partial\psi^*}{\partial x}\right]$$

$$= \frac{\hbar}{2mi}\left[A\cos[i\phi(x,t)]\cdot t\,\frac{\partial A\sin[i\phi(x,t)]}{\partial\psi}\right.$$

$$\left. - A\sin[i\phi(x,t)]\,\frac{\partial A\cos[i\phi(x,t)]\cdot t}{\partial x}\right]$$

$$= \frac{\hbar}{2mi}\left[A\cos[i\phi(x,t)]\cdot t\cdot A\cos[i\phi(x,t)]\cdot t\right.$$

$$\left. - A\sin[i\phi(x,t)\times]A\sin(i\phi(x,t)]t^2\right]$$

$$= \frac{\hbar}{2mi}A^2t^2\cos^2[i\phi(x,t)] + \sin^2[i\phi(x,t)]$$

$$= \frac{\hbar}{2mi}A^2t^2\left[\cos^2[i\phi(x,t) + \sin^2[i\phi(x,t)]\right]$$

Date
Page

$$S = \frac{\hbar^2 A^2 t^2}{2mi}$$

Q(13) A free particle of mass m is trapped in a one-dimensional box of width a. The wave function is known to be

$$\psi(x) = \frac{1}{2}\sqrt{\frac{2}{a}}\sin\left(\frac{\pi x}{a}\right) + \frac{A}{2}\sqrt{\frac{1}{a}}\sin\left(\frac{3\pi x}{a}\right)$$

$$-\frac{1}{2}\sqrt{\frac{2}{a}}\sin\left(\frac{4\pi x}{a}\right)$$ Find A.

Show that $\psi(x)$ is normalized. Which state is the most probable state.

Soln

The basic equation 1-D box.

$$\phi_n(x) = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi x}{a}\right)$$

$$\psi(x) = \frac{i}{2}\sqrt{\frac{2}{a}}\sin\left(\frac{\pi x}{a}\right) + \frac{A}{\sqrt{2}}\sqrt{\frac{2}{a}}\sin\left(\frac{3\pi x}{a}\right) -$$

$$\frac{1}{2}\sqrt{\frac{2}{a}}\sin\left(\frac{4\pi x}{a}\right)$$

$$\psi(x) = \frac{i}{2}\phi_1(x) + \frac{A}{\sqrt{2}}\phi_3(x) - \frac{1}{2}\phi_4(x)$$

Date
Page

When,

$$\left(\psi_{(x)} \cdot \psi(x)\right) = 1$$

$$\frac{1}{4} + \frac{A^2}{2} + \frac{1}{4} = 1$$

$$\frac{A^2}{2} = \frac{1}{2}$$

$$A^2 = 1$$

$$A = 1$$

$$\therefore \psi(x) = \frac{i}{2}\sqrt{\frac{2}{a}}\sin\left(\frac{\pi x}{a}\right) + \frac{1}{\sqrt{2}}\sqrt{\frac{1}{a}}\sin\left(\frac{3\pi x}{a}\right) - \frac{1}{2}\sqrt{\frac{2}{a}}\sin\left(\frac{4\pi x}{a}\right)$$

$$= \frac{i}{2}\phi_1(x) + \frac{1}{2}\phi_3(x) - \frac{1}{2}\phi_4(x).$$

Aashish

Date
Page

Q4) Let a^+ and a represent creation and annihilation operator of a harmonic oscillator having Hamiltonian H. Showed that these operators satisfy $[a, a^+] = 1$.

Soln →

Consider two non-Hermitian dimensional operators.

$$\hat{a} = \frac{1}{\sqrt{2}}(x - iP) \quad \text{and} \quad \hat{a}^+ = \frac{1}{\sqrt{2}}(x - iP)$$

Where,

$\hat{a}$ = Annihilation operator

$\hat{a}^+$ = Creation operators.

Now, $\hat{a}\hat{a}^+ \mp \frac{1}{2} = H$ —— (1)

using value of $a\,a^+$ in (1)

$$\hat{a}^+ + \hat{a} = \frac{1}{2}(x - iP) + \frac{1}{\sqrt{2}}(x + iP)$$

$$\hat{a}^+ + \hat{a} = \sqrt{2}x$$

$$x = \frac{\hat{a}^+ + \hat{a}}{\sqrt{2}}$$

by Aashich

Again,

$$\hat{a}^+ + \hat{a} = \sqrt{2}(-iP)$$

2080-11-18
→ Ashok - sodari →

Date
Page

$$p = \frac{i}{\sqrt{2}}(\hat{a} - \hat{a})$$

Now,

$$[\hat{a} + \hat{a}^{+}] = \frac{1}{2}[x + ip, x - ip]$$

$$= \frac{1}{2}\{[x, x] + [ip, x] - [x, ip] - [ip, p]\}$$

$$= \frac{1}{2}\{i[p, x] - i[x, p]\}$$

$$= \frac{1}{2}\{i(-i) - i(i)\}$$

$$= 1$$

$$\Rightarrow \hat{a} + \hat{a}^{+} - \hat{a}^{+} + \hat{a} = 1$$

$$H = \hat{a}\hat{a}^{+} - 1 + \frac{1}{2} \longrightarrow (2)$$

$$H = \hat{a}\hat{a}^{+} - \frac{1}{2} \longrightarrow (3)$$

Adding (2) and (3)

$$2H = \hat{a}^{+}\hat{a} + \hat{a}\hat{a}^{+}$$

$$H = \frac{\hat{a}^{+}\hat{a} + \hat{a}\hat{a}^{+}}{2}$$

Which is required expression of Harmonic oscillator in term of Creation and Onihilation operator.

Sumit by Ashok chand sodari
2080-11-16-0